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Some Problems in Nonlinear Analysis

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M. G. Crandall and P. H. Rabinowitz

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abstract evolution equations, Hamilton-Jacobi equations, connecting orbits, periodic & subharmonic solutions of Hamiltonian systems, nonlinear parabolic equations.

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

M. G. Crandall has been working on several problems: existence questions for abstract evolution equations, existence and uniqueness for certain classes of parabolic, Hamilton-Jacobi, and degenerate elliptic equations, and questions related to the control of partial differential equations.

P. H. Rabinowitz has been studying the existence of periodic and connecting orbits for certain nonlinear pendulum type equations. He has obtained the existence of periodic and subharmonic solutions for families of singular Hamiltonian systems.

A post-doctoral fellow, S. Angenent has developed a new approach to a class of maps of interest in the study of dynamical systems. He is also working on nonlinear parabolic equations such as arise in modelling the melting of solids and on problems in population dynamics.

(continued on reverse side)

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19. (continued)

Four predoctoral students are treating problems on periodic solutions of finite and infinite dimensional Hamiltonian systems, on variational methods to treat ordinary and partial differential equations, and on the relationship between differential games and viscosity solutions of the Hamilton-Jacobi equation.

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STATEMENT AND STATUS OF WORK

Senior Investigators:

Michael Crandall has made considerable progress on several fronts. First, jointly with Souganidis, he has completed his program of obtaining Kato's quasilinear theory by simpler arguments using methods from the theory of nonlinear semigroups. Secondly, in joint work with P.-L. Lions and T. Souganidis, he has studied certain classes of parabolic and Hamilton-Jacobi equations, and has proved the existence and uniqueness of solutions of initial-boundary value problems with singular (e.g., indentically infinite) initial data and the continuous dependence of these singular solutions as the diffusion coefficient tends to zero. This work shows how certain questions in partial differential equations motivated by the theory of large deviations can be treated in greater generality. Moreover it provides an abstract framework for the treatment as well as concrete estimates. In another paper, written jointly with R. Newcomb and Y. Tomita, Crandall extends some previous work on growth classes for Hamilton-Jacobi equations in $\mathbb{R}^n$ to obtain existence and uniqueness of viscosity solutions for degenerate elliptic problems in $\mathbb{R}^n$. Finally in a joint work with P.-L. Lions, viscosity solutions of Hamilton-Jacobi equations in infinite dimensions are studied for Hamiltonians with unbounded linear terms. This is the first major work using viscosity methods to treat the Hamilton-Jacobi equations for Hamiltonians which involve semigroup generators. It initiates the extension of viscosity theory to justify the equation of dynamic programming for the value function as it arises in the control of partial differential equations.

In current work, Crandall is attempting to extend the scope of the theory of viscosity solutions in infinite dimensions, which he has developed with P.-L. Lions. The extensions are in such directions as (a) the control of PDE's governed by self adjoint operators, (b) seeking the correct theory for the dynamic programming equation corresponding to boundary controls, and (c) unifying with the second order theory developed by Lions.

Paul Rabinowitz has mainly been working on applications of minimax techniques to problems involving Hamiltonian systems of ordinary differential equations. The first question he considered involves second order systems of "compound pendulum" type of which

$$\ddot{q} + V_q(t,q) = f(t)$$

is a special case. Here $q \in \mathbb{R}^n$, f and V are τ periodic in t, and V is periodic in q_i, $1 \leq i \leq n$. Assuming that

$$[f] \equiv \frac{1}{\tau}\int_0^\tau f(s)ds = 0 \quad ,$$

he showed that the system has at least $n + 1$ distinct families of solutions. (Note that because of the periodicity of V in the components of q, if $Q(t)$ is a solution, it is part of the family of solutions $Q(t) + (k_1T_1,\ldots,k_nT_n)$ where $(k_1,\ldots,h_n) \in \mathbb{Z}^n$.)

In work now in progress, Rabinowitz has now been studying the time

when the above result applies for any given period $\tau > 0$ but the $n + 1$ solutions may merely be equilibrium solutions corresponding to critical points of V. He gives conditions on V under which there are nonequilibrium solutions and estimates for their number. Moreover if M denotes the set of $q \in \mathbb{R}^n$ at which the global maximum of V is attained, he proves that if M consists only of isolated points there exist connecting (heteroclinic) orbits joining each point in M to other points in M.

In another direction, Rabinowitz has been studying (HS) for singular potential energies, e.g. $V(q) \sim -|q|^{-\beta}$ where $\beta > 0$. In joint work with A. Bahri, he shows that under appropriate conditions on V, for any $T > 0$, (HS) has infinitely many distinct T periodic classical solutions. This result requires that V blows up sufficiently strongly at 0, e.g. $\beta > 2$ in the above example. For potentials with less violent singularities, the existence of generalized T periodic solutions was established. Further conditions on V which enable one to determine whether these solutions are collision or non-collision orbits remains an open question. In subsequent work, Rabinowitz used different minimax arguments to obtain somewhat analogous results for forced singular systems and also found subharmonic solutions. He is now thinking about the existence of periodic solutions of Hamiltonian systems with noncompact sets of singularities such as arise in the n-body problem.

Postdoctoral Investigator:

S. Angenent is working on problems in dynamical systems and partial differential equations. Monotone twist maps are an interesting class of maps that occur in the study of dynamical systems and have attracted much interest recently. Angenent has completed a paper on monotone recurrence relations, a generalization of twist maps. He has developed new methods to treat these objects using ideas from the theory of elliptic partial differential equations such as the maximum principle. His work gives new information about maps with 0 entropy, in particular ordering properties.

Angenent is also looking at problems concerning the evolution of curves on two dimensional surfaces, e.g. curves whose motion is determined by a relation of the form

$$\nu_\perp = V(P,\underline{t},k_g) \quad .$$

Here $\nu_\perp$ is the normal velocity of the curve at the point P, $\underline{t}$ is its unit vector at that point, and k_g is its geodesic curvature. The function V is assumed to be known. The goal is to find the weakest conditions on V which guarantee that the initial value problem is well posed. Angenent has obtained some results for special cases in earlier published work.

The above is a simpler version of more complicated problems that Angenent is studying jointly with M. Gurtin. They are considering mathematical models for the melting of a solid or more general phase transitions in which the heat conductivity is assumed to be infinite and the most important physical phenomena occur at the (free) boundary of the solid, $\partial\Omega$. Here Ω is the

$$\beta(\theta)v_{\perp} = (f(\theta) + f''(\theta))k - F \quad .$$

Lastly, jointly with D. Aronson, Angenent is working on a degenerate parabolic initial value problem:

$$u_t = (u^m)_{xx} \; tu(1-u)(u-a) \qquad -L < x < L, \; t > 0$$

$$u(\pm L,t) \equiv 0, \; u(\theta,0) \quad \text{given} \quad .$$

Here $m > 1$ is a constant. This equation which was originally proposed by Gurtin and McCamy models the evolution of a population which disperses to avoid crowding. The questions that are being studied are the existence and behavior of connecting orbits between steady states of this problem and how, in particular, their behavior differs from the corresponding orbits for the well understood non-degenerate case $m = 1$.

Predoctoral Investigators:

P. Felmer is beginning to work on a dissertation. He is studying generalized versions of the Birkhoff-Lewis Theorem on subharmonic solutions of Hamiltonian systems and also has some generalizations of work of Conley and Zehnder.

Y. Long completed his thesis which was on periodic solutions of finite and infinite dimensional Hamiltonian systems, the latter involving nonlinear wave and Schroedinger equations. He has developed new technical machinery to treat Hamiltonians which grow more rapidly than quadratically at infinity and significantly generalized known results. He won an award from our graduate school for an outstanding thesis.

R. Newcomb is working on the connection between the general existence and uniqueness theory for viscosity solutions of Hamilton-Jacobi equations and the theory of differential games. He has made considerable progress in unifying and clarifying these areas and will obtain his Ph.D. in August 1988.

E. Silva is developing various variational minimax tools to treat nonlinear elliptic PDE's and ODE's. He has obtained several generalizations of the so-called Saddle-Point Theorem and applications to differential equations.

PUBLICATIONS

M. G. Crandall

On nonlinear evolution equations, (with P. Souganidis), to appear in Nonlin. Anal. Th. Meth. Appl. (ms. 26 pages).

Maximal solutions and universal bounds for some quasilinear evolution equations of parabolic type (with P. L. Lions and P. Souganidis), to

Existence and uniquenss of viscosity solutions of degenerate quasilinear elliptic equations in $\mathbb{R}^n$ (with R. Newcomb and Y. Tomita), submitted, (ms. 19 pages).

Viscosity solutions of Hamilton-Jacobi equations in infinite dimensions. Part IV. Hamiltonians with unbounded linear terms, (with P. L. Lions), submitted, (ms. 50 pages).

Paul H. Rabinowitz

On a class of functionals Invariant under a Z^n action, to appear, Trans. A.M.S.

A minimax method for a class of Hamiltonian systems with singular potentials (with A. Bahri), to appear J. Functinal Analysis.

Periodic solutions for some forced singular Hamiltonian systems (to appear in volume in honor of Jürgen Moser's 60th birthday).

Periodic and heteorclinic orbits for a class of periodic Hamiltonian systems (in progress).

S. Angenent

Monotone recurrence relations, their Birkhoff orbits and topological entropy, to appear in Ergodic Theory and Dynamical Systems.

Parabolic equations for isometrically immersed curves on surfaces (in progress).

The motion of the boundary of a melting solid (with M. Gurtin), (in progress)

A model from population dynamics (with D. G. Aronson), (in progress).

Y. Long

Multiple solutions of perturbed superquadratic second order Hamiltonian systems (to appear Trans. AMS).

PARTICIPATING PERSONNEL

Principle Investigators (unsalaried)

M. G. Crandall

P. H. Rabinowitz

Postdoctoral fellow

Predoctoral fellows

P. Felmer 5/87 - 7/87

Y. Long 5/87 - 7/87
Ph.D. 12/87 "Periodic solutions of perturbed superquadratic Hamiltonian systems"

R. Newcomb 8/86 - 7/88
Ph.D. 8/88 "Existence and correspondence of value functions and viscosity solutins of Hamilton-Jacobi equations"

E. Silva 1/88 - 7/88

INTERACTIONS: INVITED LECTURES

M. G. Crandall

Ecole Normale Superieure Paris - June 1987
Maximal solutions and universal bounds for some nonlinear evolution problems.

Autonoma University of Madrid - June 1987
Continuous dependence and generalized solutions for some degeneral nonlinear diffusion problems.

Brown University - Aug. 1987
International Workshop on PDE's in Stochastic Control and Differential Games - Viscosity solutions of Hamilton-Jacobi equation in infinite dimensions.

University of Besancon - Sept.-Oct. 1987
Viscosity solutions of Hamilton-Jacobi equations in infinite dimensions (2 lectures), Maximal solutions and universal bounds for some nonlinear evolution problems.

University of Paris VI - Oct. 1987
Continuous dependence and generalized solutions for some degenerate nonlinear diffusion problems.

Scuola Normale Superiore di Pisa - Nov. 1987
Viscosity solutions of Hamilton-Jacobi equations in infinite dimensions.

College de France - Nov. 1987
Viscosity solutions of Hamilton-Jacobi equations in infinite dimensions.

University of Paris IX - Dec. 1987
A new framework for some nonlinear evolution problems.

University of Nancy - Dec. 1987
Viscosity solutions of Hamilton-Jacobi equations in infinite dimensions.

Brown University - March 1988
(conference in honor of W. H. Fleming) Theory of viscosity solutions.

Institute for Advanced Study -2 seminars (Princeton) - March 1988
Viscosity solutions of Hamilton-Jacobi equations in infinite dimensions.

University of Maryland - April 1988
(conference in honor of A. Douglis) Generalized solutions of Hamilton-Jacobi equations.

P. H. Rabinowitz

University of Miami - Feb. 1988
Periodic solutions to a class of nonlinear pendulum equations.

Society for Natural Philosophy (Baltimore) - March 1988
Periodic solutions of Hamiltonian systems.

Courant Institute - March 1988
Periodic solutions for a class of singular Hamiltonian systems.

Institute for Advanced Studies (Princeton) - April 1988
Periodic solutions of Hamiltonian systems & Periodic solutions for a class of singular Hamiltonian systems.

Rutgers University - April 1988
Periodic solutions to a class of nonlinear pendulum equations.

Colloquium Lions (Paris) - June 1988
Periodic solutions of Singular Hamiltonian systems.

International Symposium on Variational Problems (Paris) - June 1988
Periodic solutions of Singular Hamiltonian systems.

S. Angenent

University of Leiden, the Netherlands - Sept. 1987
On the zeroset of a solution of a linear parabolic equation.

Dynamical Systems Seminar, Cal. Tech. - Jan. 1988

AMS Meeting, Maryland - April 1988

Parabolic equations that define local semiflows on the space of curves on a surface.

Univ. of Minnesota - May 1988

Analysis Seminar - Parabolic equations for curves on surfaces.

Dynamical Systems Seminar - Twistmaps and topological entropy.

Univ. of California, San Diego - June 1988

Hamilton's Seminar - Parabolic equations for curves on surfaces.